is appropriately arranged as 0.2675 m. For a maximum angle of oscillation θ_0 below 40 deg, the nonlinear effect on the oscillation period is minimal (less than 0.1% difference from the exact value)[5]; thus Eq. (14) can be appropriately applied to obtain the J_{xx}. For this test, the average period of 50 cycles $T = 1.197$ s is used in Eq. (14) to obtain J_{xx} as 1.881 kg-m^2, which is less than 1% difference from the accurately calculated value of 1.8647 kg-m^2. On the other hand, if Eq. (1) instead of Eq. (14) is employed for this calculation, it can be easily seen that the error involved would be around 15% below the true J_{xx} value of the tested cylinder. Thus Eq. (14) shows a significant improvement in accuracy over the traditional Eq. (1).

Conclusion

Although the minor modification of the support configuration for an oscillating test seems negligible, it can actually degrade the accuracy of J_{xx} seriously. Since the knife-edge support for an oscillation test cannot be fully realized, it would be necessary to reanalyze the theoretical side to accommodate the practical experimental difficulties, otherwise noticeable errors will occur. Furthermore, for obtaining even higher accuracy of J_{xx} at relatively large test oscillation angles, it would be feasible to take into consideration the nonlinear effect on the oscillation period.[5] Incidentally, the condition for the minimum period of such a special compound pendulum has been derived. It is closely related to the radius of gyration of the test missile; in conjunction with other test parameters, these data are useful in adjusting the center-to-center distance h to optimize the test conditions.

On the other hand, with some advantages and disadvantages, other test methods may also be applied to obtain J_{xx}. For example, a missile may be assembled as a compound pendulum with appropriate fixture and supporting roller bearings; then the simple torsional dynamics equation can be used to obtain J_{xx} of the whole oscillating system and subsequently the J_{xx} of the test missile.

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New Design Formulas for a Flex-Bearing Joint

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Introduction

FLEX bearing is a nonrigid, pressure-tight connection between the rocket motor and movable nozzle that allows the movable nozzle to deflect in a specified direction. Solid propellant rocket stages are generally equipped with these flexible joints to provide thrust vector control. These joints usually consist of several conical/spherical rings of elastomeric material alternating with conical/spherical reinforcements as shown in Fig. 1.

Akiba et al.[1] presented the development and test results of spherical flex bearings. Design, analysis, and testing details of flex bearings for various configurations of reinforcements and elastomers are available in Refs. 2–5. The design formulas for the compressive hoop stresses in the reinforcements, proposed in Refs. 2 and 5 are derived from the test results of joints varying from 19.3 to 56 cm in diameter and hence are valid only in this range. Based on these formulas, a flex-bearing joint was designed. Subsequently, the geometrically nonlinear finite element analysis was carried out using the general-purpose program MARC. The analysis results were found to be much different compared with the design formulas assessment.[2–5] Later, the test results confirmed the analysis predictions. Furthermore, the design formulas[2,5] for the compressive hoop stresses are seen to be highly sensitive to reinforcement thickness, and the stresses so obtained are linearly proportional to the motor pressure, whereas the analysis and test results did not show this trend. These inadequacies in the existing formulas motivated the authors to develop refined formulas to have accurate predictions of the compressive hoop stresses at the inner radii of the reinforcements. The refined formulas are developed based on the analyses and test results of the following configurations of flex-bearing joints: 1) six elastomer pads (of natural rubber formulation), each of 3.0 mm thickness, and six 15-cdv-6 steel reinforcements, each of 3.0 mm thickness; and 2) six elastomer pads, each of 3.0 mm thickness, and the thickness of 15-cdv-6 steel reinforcements (in millimeters) is varied as 4.5 /5.5 /5.5 /5.5 /5.5 /4.5.

The effectiveness of these formulas are shown through test results. Since the proposed formulas are an improvement over the existing formulas (Refs. 2 and 5), the applicability and validity range of the formulas remain the same, that is, joints having diameters within 19.3–56 cm.

Existing Design Formulas

The stresses in the reinforcements are tensile hoop on the outer radius and compressive hoop stress on the inner radius due to motor pressure and actuation loads.[2,5] In Ref. 5 the following formulas are suggested to compute the compressive hoop stress due to motor pressure and actuation loads:

$$\sigma_P = \frac{4087}{n-1} P_C K_R \Omega \tag{1}$$

$$\sigma_V = \frac{43{,}950}{n-1} \theta K_R \Omega \tag{2}$$

with

$$\Omega = \frac{R_P^{2.4}\cos\beta}{3283 t_R^3 + t_R \cos^2\beta[R_P^2(\beta_2 - \beta_1)^2 - 3283 t_R^2]}$$

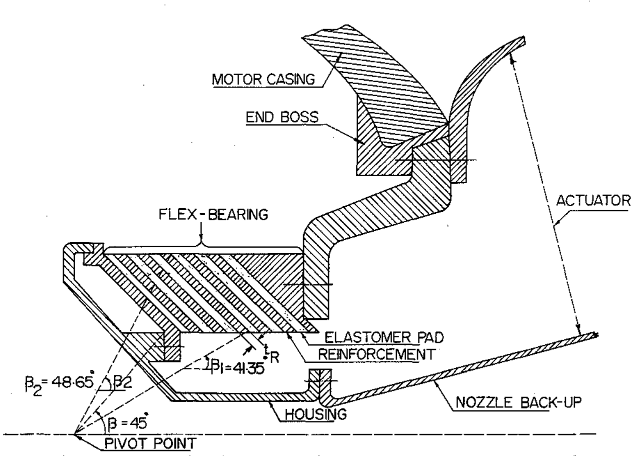

Fig. 1 Typical flex bearing in position with other subassemblies.

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Table 1 Compressive hoop stress in the middle reinforcement due to motor/chamber pressure, kg/mm^2

Flex-bearing	Pressure, kg/cm^2	Ref. 5	Present formulas	FEM[a]	Test
Configuration I	15.21	54.92	32.98	32.04	33.36
	30.42	109.84	61.07	56.61	60.60
	45.64	164.80	84.28	81.55	88.91
	60.85	219.72	104.60	102.33	113.04
	71.50	258.17	120.56	115.71	119.42
Configuration II	15.21	29.61	23.94	23.01	22.61
	30.42	59.21	44.33	41.44	44.10
	45.64	88.83	61.19	58.10	61.46
	60.85	118.44	75.92	73.52	77.90
	71.50	139.17	87.57	81.90	88.21

[a]Finite element method.

Table 2 Compressive hoop stress in the middle reinforcement due to 3-deg actuation

	Configuration 1			Configuration 2		
Pressure, ksc	Ref. 5	Present formula	Test	Ref. 5	Present formula	Test
10.00	8.2	16.59	15.48	4.42	12.06	8.10
20.00	8.2	18.10	21.02	4.42	13.15	12.40
40.00	8.2	21.28	20.98	4.42	15.50	16.20
47.00	8.2	24.15	29.42	4.42	17.47	19.60

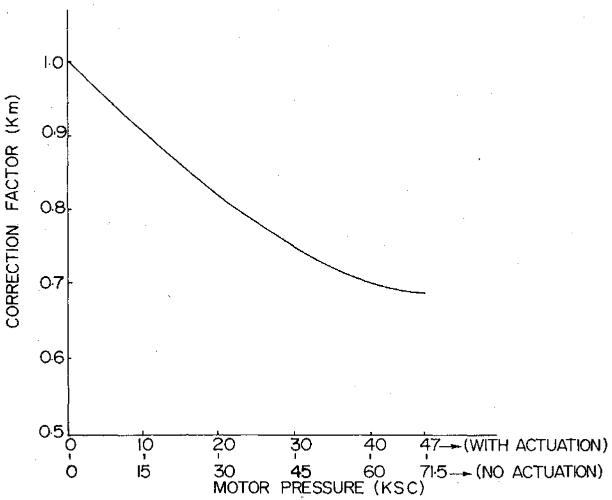

Fig. 2 Variation of correction factor K_M with motor pressure (kg/cm^2).

where

σ_P = compressive hoop stress due to motor pressure (psi)
σ_V = maximum compressive hoop stress due to actuation load only (psi)
P_C = motor pressure
K_R = correction factors[5]
n = number of elastomer pads
R_P = pivot radius (in.) Fig. 1
t_R = reinforcement thickness (in.)
β_2 and β_2 = outer and inner joint angles (deg) (Fig. 1)
$\beta_2 = (\beta_2 + \beta_1)/2.0$
θ = vector angle (deg)

The total hoop stress σ_T in the reinforcements due to motor pressure and vectoring can be obtained by simply adding σ_P and σ_V, that is,

$$\sigma_T = \sigma_P + \sigma_V \tag{3}$$

It must be remarked here that the expression for Ω, which is supposed to be dimensionless, is not so.

Proposed Design Formulas

The available design formulas (1-3) are inadequate for the design of flex-bearing joints having conical reinforcements and elastomer pads. The following design formulas to compute the compressive hoop stresses at the inner radius of the reinforcement due to motor pressure and actuation loads are developed based on the analyses results and are verified through test data:

$$\sigma_P = \frac{4370}{n-1} P_C K_R K_M \Omega \tag{4}$$

and

$$\sigma_V = \frac{184{,}516}{n - 1K_M} \theta K_R \Omega \tag{5}$$

with

$$\Omega = \frac{R_P^{2.4}}{3283 t_R^{2.4} \sin^2\beta + t_R^{0.5} R_P^{1.9} (\beta_2 - \beta_1)^2 \cos^2\beta}$$

where K_M is a correction factor based on the motor pressure (Fig. 2), and other notations remain as described earlier.

On comparing Eqs. (1) and (2) with Eqs. (4) and (5), it may be observed that an additional factor K_M based on motor pressure has been brought in, the expression for Ω has been made a dimensionless parameter, and the constant multiplying factors have also been changed to yield accurate predictions of compressive hoop stresses.

Results and Discussions

The variation of compressive hoop stress with motor pressure for two configurations of flex bearings are presented in Table 1. It may be observed that design formulas given in Ref. 5 lead to very high stresses, leaving the designer with no other option but to use thicker reinforcements, paying a heavy weight penalty. Furthermore, on comparing the hoop stresses obtained from these formulas[5] for the two configurations, one can conclude that the stresses are highly sensitive to reinforcement thickness. At the same time, a look at the analyses and test results contradicts this conclusion. The present design formulas give fairly accurate estimation of stresses when compared with analyses and test results.

Table 2 shows the variation of compressive hoop stress due to 3-deg actuation at various motor/chamber pressures. The compressive hoop stress values presented in this table are obtained by subtracting hoop stress values due to motor pressure alone (that is, without actuation). The proposed formula, Eq. (5), again gives a good estimate of hoop stress due to actuation, whereas those available in Ref. 5 can be seen to yield very low stresses when compared with test results.

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